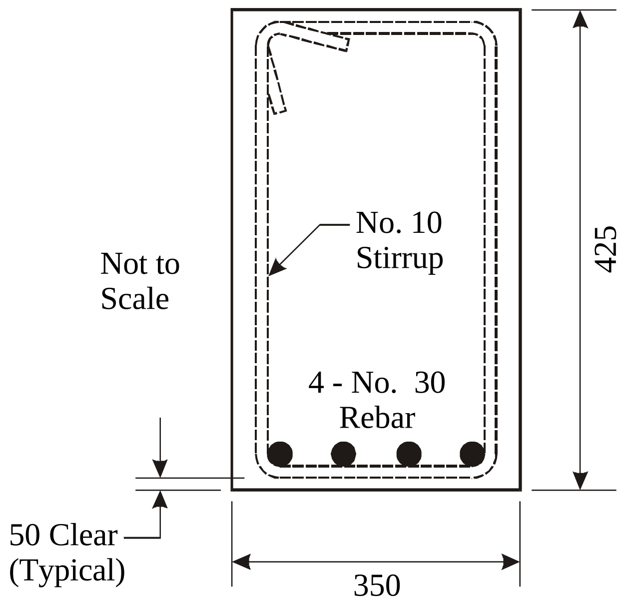
$MPa := 10^6 \cdot Pa$ $\quad kN := 10^3 \cdot N$ $\quad E_s := 200000 \cdot MPa$ $\quad \phi_c := 0.6$ $\quad \phi_s := 0.85$

Question 1



Beam dimensions:

$h := 425 \cdot mm$

$b := 350 \cdot mm$

$cc := 50 \cdot mm$

$d_{st} := 10 \cdot mm$ (Stirrup dia.)

Tension steel:

$d_b := 30 \cdot mm$ $\quad n_b := 4$

$A_{bar} := 700 \cdot mm^2$

$A_s := n_b \cdot A_{bar}$

$A_s = 2800\,mm^2$

$f_y := 400 \cdot MPa$

- Effective depth: $d := h - cc - d_{st} - \frac{d_b}{2}$ $\quad d = 350\,mm$

Whitney stress block parameters: $f'_c := 25 \cdot MPa$ $\quad \varepsilon_{cu} := 0.0035$

$$\alpha_1 := \begin{cases} 0.85 - 0.0015 \cdot \frac{f'_c}{MPa} & \text{if } 0.85 - 0.0015 \cdot \frac{f'_c}{MPa} \ge 0.67 \\ 0.67 & \text{otherwise} \end{cases} \qquad \alpha_1 = 0.813$$

$$\beta_1 := \begin{cases} 0.97 - 0.0025 \cdot \frac{f'_c}{MPa} & \text{if } 0.97 - 0.0025 \cdot \frac{f'_c}{MPa} \ge 0.67 \\ 0.67 & \text{otherwise} \end{cases} \qquad \beta_1 = 0.907$$

(a) Factored Moment Capacity:

- Strain and stress in reinforcing steel: $\varepsilon_s = \varepsilon_{cu} \cdot \left(\frac{d - c}{c}\right)$ $\quad f_s = E_s \cdot \varepsilon_s = E_s \cdot \left[\varepsilon_{cu} \cdot \left(\frac{d - c}{c}\right)\right]$

- Depth of compression stress block: $\Sigma F_x = 0$ $C_c = T$

$$C_c = (\phi_c \cdot \alpha_1 \cdot f'_c)\cdot(a\cdot b) = (\phi_c \cdot \alpha_1 \cdot f'_c)\cdot(\beta_1 \cdot c \cdot b) \qquad T = \phi_s \cdot A_s \cdot f_s = \phi_s \cdot A_s \cdot \left[E_s \cdot \left[\varepsilon_{cu} \cdot \left(\frac{d-c}{c}\right)\right]\right]$$

$$(\phi_c \cdot \alpha_1 \cdot f'_c)\cdot(\beta_1 \cdot c \cdot b) = \phi_s \cdot A_s \cdot \left[E_s \cdot \left[\varepsilon_{cu} \cdot \left(\frac{d-c}{c}\right)\right]\right]$$

$$(\phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b)\cdot c^2 + (\phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu})\cdot c - \phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu} \cdot d = 0$$

where: $(\phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1 \cdot b) = 0.004\,m^2 \dfrac{MPa}{mm}$ $(\phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu}) = 1.666 \times 10^6\,N$

$\phi_s \cdot A_s \cdot E_s \cdot \varepsilon_{cu} \cdot d = 5.831 \times 10^8\,N \cdot mm$

- Solving:

$c = 228.588\,mm$ $a := \beta_1 \cdot c$ $a = 207.444\,mm$

- Check equilibrium: $\varepsilon_s := \varepsilon_{cu} \cdot \left(\dfrac{d-c}{c}\right)$ $\varepsilon_s = 0.00186$ $\varepsilon_y := \dfrac{f_y}{E_s}$ $\dfrac{\varepsilon_s}{\varepsilon_y} = 0.929$

$$f_s := \begin{cases} E_s \cdot \varepsilon_s & \text{if } \varepsilon_s \le \varepsilon_y \\ f_y & \text{otherwise} \end{cases}$$

$f_s = 371.797\,MPa$

Therefore, steel doesn't yield.

$T := \phi_s \cdot A_s \cdot f_s$ $T = 884.877\,kN$

$C_c := (\phi_c \cdot \alpha_1 \cdot f'_c)\cdot(a \cdot b)$ $C_c = 884.877\,kN$ - OK

- Ultimate moment resistance:

$M_r := T \cdot \left(d - \dfrac{a}{2}\right)$ $M_r = 217.926\,kN \cdot m$

(b) Compressive steel to produce a balanced failure condition

Location of neutral axis:

$\dfrac{\varepsilon_{cu}}{c} = \dfrac{\varepsilon_y}{d-c}$ $c := \varepsilon_{cu} \cdot \dfrac{d}{(\varepsilon_{cu} + \varepsilon_y)}$ $c = 222.727\,mm$

$a := \beta_1 \cdot c$ $a = 202.125\,mm$

$$C_c := (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (a \cdot b) \qquad C_c = 862.189\,\mathrm{kN}$$

Tension steel area to balance compression in concrete (knowing that steel yields):

$$T_2 = C_c \qquad \phi_s \cdot A_{s2} \cdot f_y = (\phi_c \cdot \alpha_1 \cdot f'_c) \cdot (a \cdot b) \qquad A_{s2} := \phi_c \cdot \alpha_1 \cdot f'_c \cdot a \cdot \frac{b}{(\phi_s \cdot f_y)}$$

$$A_{s2} = 2535.851\,\mathrm{mm}^2$$

Tension steel area to be balance by compression steel:

$$A_{s1} := A_s - A_{s2} \qquad A_{s1} = 264.149\,\mathrm{mm}^2$$

$$(\phi_s \cdot f_y - \phi_c \cdot \alpha_1 \cdot f'_c) \cdot A'_s = \phi_s \cdot f_y \cdot A_{s1}$$

$$A'_s := \phi_s \cdot f_y \cdot \frac{A_{s1}}{(\phi_s \cdot f_y - \phi_c \cdot \alpha_1 \cdot f'_c)} \qquad A'_s = 273.969\,\mathrm{mm}^2$$

check yielding: Assume No. 15 bars $\quad d'_b := 15 \cdot \mathrm{mm} \qquad d' := cc + d_{st} + \frac{d'_b}{2}$

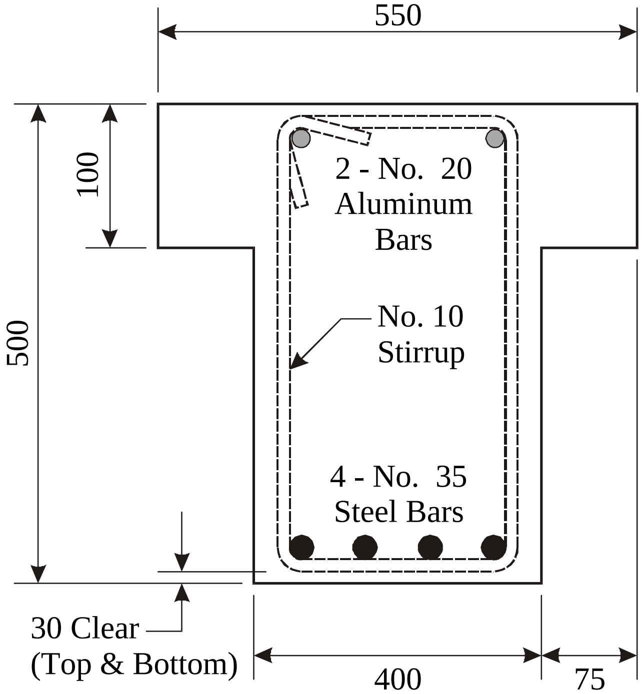
$$d' = 67.5\,\mathrm{mm} \qquad \frac{\varepsilon_{cu}}{c} = \frac{\varepsilon'_s}{c - d'} \qquad \varepsilon'_s := \frac{-\varepsilon_{cu}}{c} \cdot (-c + d') \qquad \varepsilon'_s = 0.00244$$

$$\frac{\varepsilon'_s}{\varepsilon_y} = 1.22$$

Therefore, compression steel yields.

$MPa := 10^6 \cdot Pa$ $\quad kN := 10^3 \cdot N$ $\quad E_s := 200000 \cdot MPa$

Question 2:



Beam dimensions:

$b_F := 550 \cdot mm$

$h_F := 100 \cdot mm$

$b_w := 400 \cdot mm$

$h := 500 \cdot mm$

$cc := 30 \cdot mm$

Tension reinforcement:

$d_b := 35 \cdot mm \qquad n_b := 4$

$A_{s\_bar} := 1000 \cdot mm^2$

$A_s := n_b \cdot A_{s\_bar}$

$A_s = 4000\,mm^2$

$d := h - cc - 10 \cdot mm - \dfrac{d_b}{2}$

$d = 442.5\,mm$

$f_y := 425 \cdot MPa$

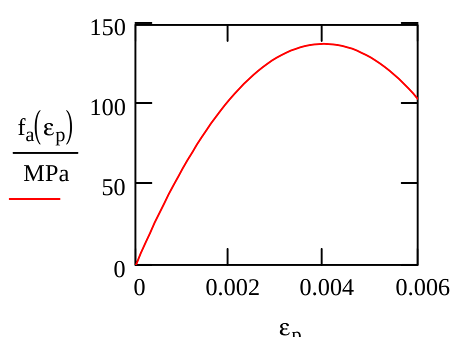
Aluminum bars:

$d_a := 20 \cdot mm \qquad n_a := 2 \qquad A_{bar\_a} := 310 \cdot mm^2 \qquad A_a := n_a \cdot A_{bar\_a} \qquad A_a = 620\,mm^2$

$d' := cc + 10 \cdot mm + \dfrac{d_a}{2} \qquad d' = 50\,mm$

Stress-strain relationship: $\quad f_a(\varepsilon_a) := \left(69 \cdot 10^3 \cdot \varepsilon_a - 8.62 \cdot 10^6 \cdot \varepsilon_a^{\,2}\right) \cdot MPa$

- Strain range for plotting: $\quad \varepsilon_p := 0, 0.0001 .. 0.006$



P

Whitney stress block parameters: $f'_c := 30 \cdot MPa$ $\quad \varepsilon_{cu} := 0.0035$

$$\alpha_1 := \begin{cases} 0.85 - 0.0015 \cdot \dfrac{f'_c}{MPa} & \text{if } 0.85 - 0.0015 \cdot \dfrac{f'_c}{MPa} \ge 0.67 \\ 0.67 & \text{otherwise} \end{cases} \qquad \alpha_1 = 0.805$$

$$\beta_1 := \begin{cases} 0.97 - 0.0025 \cdot \dfrac{f'_c}{MPa} & \text{if } 0.97 - 0.0025 \cdot \dfrac{f'_c}{MPa} \ge 0.67 \\ 0.67 & \text{otherwise} \end{cases} \qquad \beta_1 = 0.895$$

Neutral axis location: $\varepsilon_a := 0.00232$

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_a}{c - d'} \qquad c := \varepsilon_{cu} \cdot \frac{d'}{(\varepsilon_{cu} - \varepsilon_a)} \qquad c = 148.31\,mm$$

- Depth of concrete compressive stress block: $a := \beta_1 \cdot c$ $\quad a = 132.733\,mm$

Stress in Aluminum bars (in compression): $f'_a := f_a(\varepsilon_a)$ $\quad f'_a = 113.684\,MPa$

Nominal moment capacity:

Compression in aluminum bars (including allowance for hole):

$$C_a := (f'_a - \alpha_1 \cdot f'_c) \cdot A_a \qquad C_a = 55.511\,kN$$

$$M_a := C_a \cdot (d - d') \qquad M_a = 21.788\,kN \cdot m$$

Compression in concrete flange: $A_{cF} := (b_F - b_w) \cdot h_F$ $\quad A_{cF} = 15000\,mm^2$

$$C_{cF} := (\alpha_1 \cdot f'_c) \cdot A_{cF} \qquad C_{cF} = 362.25\,kN$$

$$M_{cF} := C_{cF} \cdot \left(d - \frac{h_F}{2}\right) \qquad M_{cF} = 142.183\,kN \cdot m$$

- Moment arm: $\left(d - \frac{h_F}{2}\right) = 392.5\,mm$

Compression in concrete web: $A_{cw} := b_w \cdot a$ $\quad A_{cw} = 53093.22\,mm^2$

$$C_{cw} := (\alpha_1 \cdot f'_c) \cdot A_{cw} \qquad C_{cw} = 1282.201\,kN$$

$$M_{cw} := C_{cw} \cdot \left(d - \frac{a}{2}\right) \qquad M_{cw} = 482.279\,kN \cdot m$$

- Moment arm: $\left(d - \frac{a}{2}\right) = 376.133\,mm$

Total nominal moment capacity: $M_n := M_a + M_{cF} + M_{cw}$

$M_n = 646.25\,kN \cdot m$

Check ΣFx=0:

Stress in bars:

- Tension steel bars: $\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_s}{d-c}$ $\varepsilon_s := \varepsilon_{cu} \cdot \frac{(d-c)}{c}$ $\varepsilon_s = 0.00694$

$\varepsilon_y := \frac{f_y}{E_s}$ $\varepsilon_y = 0.00213$ $\frac{\varepsilon_s}{\varepsilon_y} = 3.267$ Therefore, steel yields.

$$f_s := \begin{cases} E_s \cdot \varepsilon_s & \text{if } \varepsilon_s \le \varepsilon_y \\ f_y & \text{otherwise} \end{cases}$$

$f_s = 425\,MPa$

$T := f_s \cdot A_s$ $T = 1700\,kN$

$C := C_a + C_{cF} + C_{cw}$ $C = 1699.962\,kN$

$MPa := 10^{6} \cdot Pa$ $\quad kN := 10^{3} \cdot N$ $\quad E_s := 200000 \cdot MPa$ $\quad \phi_c := 0.6$ $\quad \phi_s := 0.85$

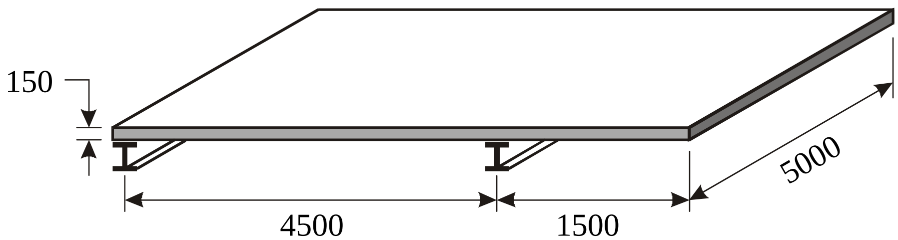
$kPa := 10^{3} \cdot Pa$

Question 3:

Slab dimensions: $L_1 := 4500 \cdot mm$ $\quad L_2 := 1500 \cdot mm$ $\quad h_s := 150 \cdot mm$ $\quad cc := 20 \cdot mm$

Loading: $q_D := 1.25 \cdot kPa$ $\quad q_L := 4.8 \cdot kPa$ $\quad \gamma_c := 2400 \cdot \frac{kg}{m^3}$

Material: $f'_c := 30 \cdot MPa$ $\quad f_y := 400 \cdot MPa$ $\quad \varepsilon_{cu} := 0.0035$



Whitney stress block parameters:

$$\alpha_1 := \begin{cases} 0.85 - 0.0015 \cdot \frac{f'_c}{MPa} & \text{if } 0.85 - 0.0015 \cdot \frac{f'_c}{MPa} \ge 0.67 \\ 0.67 & \text{otherwise} \end{cases} \qquad \alpha_1 = 0.805$$

$$\beta_1 := \begin{cases} 0.97 - 0.0025 \cdot \frac{f'_c}{MPa} & \text{if } 0.97 - 0.0025 \cdot \frac{f'_c}{MPa} \ge 0.67 \\ 0.67 & \text{otherwise} \end{cases} \qquad \beta_1 = 0.895$$

- Effective depth: Assume $\quad d_b := 15 \cdot mm$

$$d := h_s - cc - \frac{d_b}{2} \qquad d = 122.5\,mm$$

Factored load effects: Unit design strip $\quad b := 1000 \cdot mm$ $\quad \alpha_D := 1.25$ $\quad \alpha_L := 1.5$

Slab self weight: $\quad w_{Dsw} := (h_s \cdot b) \cdot (\gamma_c \cdot g)$ $\quad w_{Dsw} = 3.53 \frac{kN}{m}$

Superimposed dead load: $\quad w_D := q_D \cdot b$ $\quad w_D = 1.25 \frac{kN}{m}$

Superimposed live load: $w_L := q_L \cdot b$ $\qquad w_L = 4.8\,\dfrac{kN}{m}$

Total factored load on slab: $w_f := \alpha_D \cdot (w_{Dsw} + w_D) + \alpha_L \cdot w_L$ $\qquad w_f = 13.175\,\dfrac{kN}{m}$

Negative moment over cantilevered support: $M_f := -(w_f \cdot L_2) \cdot \left(\dfrac{L_2}{2}\right)$ $\qquad M_f = -14.822\,kN \cdot m$

Set: $M_r := -M_f$

Normalised moment: $K_r := \dfrac{M_r}{b \cdot d^2}$ $\qquad K_r = 0.988\,MPa$

- Required steel areas for strength:

$$K_r = \phi_s \cdot \rho \cdot f_y \cdot \left(1 - \frac{\phi_s \cdot \rho \cdot f_y}{2 \cdot \phi_c \cdot \alpha_1 \cdot f'_c}\right)$$

$$\rho_{req}(K_r) := \frac{\left[\phi_c \cdot \alpha_1 \cdot f'_c - \left(\phi_c^{\,2} \cdot \alpha_1^{\,2} \cdot f'^{\,2}_c - 2 \cdot K_r \cdot \phi_c \cdot \alpha_1 \cdot f'_c\right)^{\left(\frac{1}{2}\right)}\right]}{(f_y \cdot \phi_s)}$$

$\rho_{req}(K_r) = 0.00301$

$A_{s\_neg} := \rho_{req}(K_r) \cdot b \cdot d$ $\qquad A_{s\_neg} = 368.915\,mm^2$ $\qquad$ per m width

Try No. 10 bars: $A_{bar} := 100 \cdot mm^2$

$n_{bar} := \dfrac{A_{s\_neg}}{A_{bar}}$ $\qquad n_{bar} = 3.689$ $\qquad$ per m width

Spacing: $s_{bar} := \dfrac{b}{n_{bar}}$ $\qquad s_{bar} = 271.065\,mm$

Say use No. 10 bars at 250 mm o.c. $\qquad s_{bar} := 250 \cdot mm$

Check maximum spacing:

$s_{max} := \begin{cases} 3 \cdot h_s & \text{if } 3 \cdot h_s \le 500 \cdot mm \\ (500 \cdot mm) & \text{otherwise} \end{cases}$ $\qquad s_{max} = 450\,mm$ $\qquad$ > s OK

Check yielding: $n_{bar} := \frac{b}{s_{bar}}$ $n_{bar} = 4$ per m

$A_{s\_act} := n_{bar} \cdot A_{bar}$ $A_{s\_act} = 400\,mm^2$

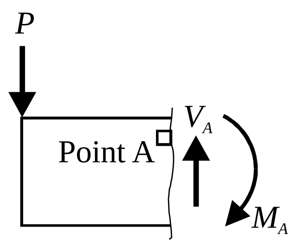
$\rho_{act} := \frac{A_{s\_act}}{b \cdot d}$ $\rho_{act} = 0.00327$

- Balanced Reinforcement Ratio:

$$\rho_b := \frac{\phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1}{\phi_s \cdot f_y} \cdot \left(\frac{700}{700 + \frac{f_y}{MPa}} \right) \qquad \rho_b = 0.024$$

$\frac{\rho_{act}}{\rho_b} = 0.135$ Therefore, the slab is under-reinforced and Table 2.1 is valid

Question 4 a):



Stress State | Mohr's Circle | Crack Orientation

Point 2: